Closing Tues: 2.8
Closing next Thurs: 3.1-2
Closing next Fri: 3.3 (last before Exam 1)

2.8 Derivative Function (continued)

Sometimes "slope of tangent" doesn't make sense.

Def'n: We say a function, $y = f(x)$, is **<u>differentiable</u>** at $x = a$ if

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \text{'a number'}.$$

Otherwise it is not differentiable at a.

To be differentiable:

1. It must be defined at x = a.
2. It must be continuous at x = a.
3. Same "slope" from "both sides".

Examples (try to sketch these):

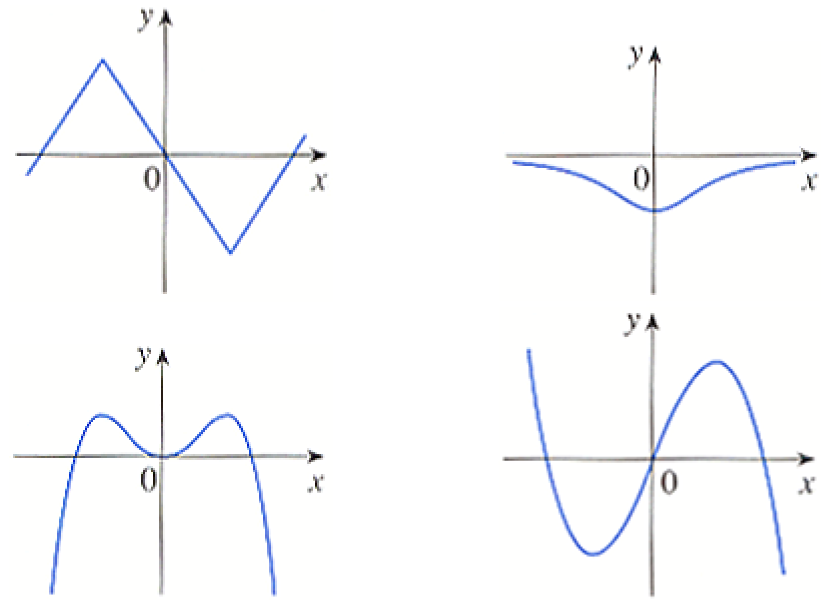
1. $f(x) = \frac{1}{x-3}$
2. $g(x) = \begin{cases} 2x - 1 & \text{, if } x < 2; \\ x^2 & \text{, if } x \geq 2. \end{cases}$
3. $k(x) = |x|$
4. $j(x) = x^{1/3}$

Matching Derivative Graphs

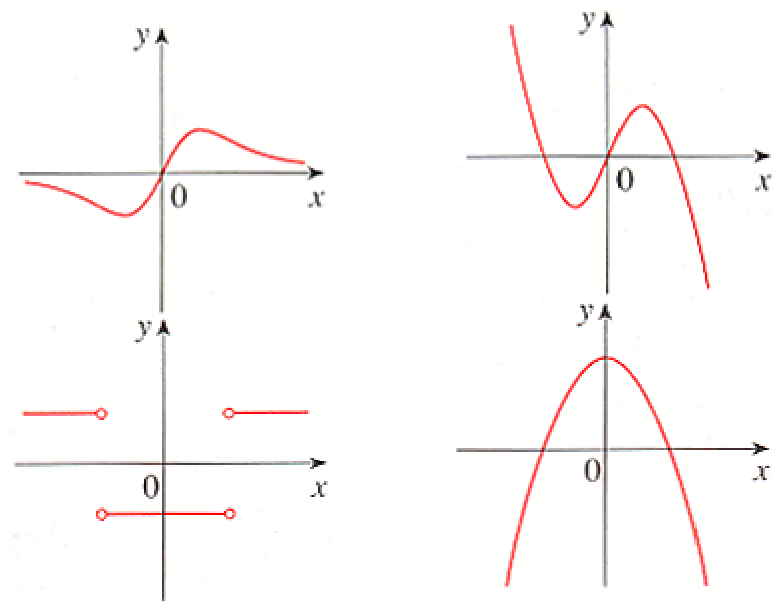
$y = f(x)$	$y = f'(x)$
horizontal tangent	zero (crosses x-axis)
increasing (uphill)	positive (above *x*-axis)
decreasing (downhill)	negative (below *x*-axis)
not differentiable	undefined

From homework: Match the graphs

Original Functions



Derivatives



Derivative Notation

Last time, we found

if $f(x) = 2x^2 - 3x,$

then $f'(x) = 4x - 3.$

Other ways to write this include:

$$y' = 4x - 3$$

$$\frac{dy}{dx} = 4x - 3$$

$$\frac{d}{dx}(2x^2 - 3x) = 4x - 3.$$

2nd Derivatives

Later we will also discuss:

$$f''(x) = y'' = \frac{d(dy/dx)}{dx} = \frac{d^2y}{dx^2}$$

Example:

if $y = f(x) = 2x^2 - 3x,$

then $y' = f'(x) = 4x - 3$

and $y'' = f''(x) = 4$

which can also be written as

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(4x - 3) = 4$$

3.1/3.2 Intro to Derivative Rules

Some Basic Limit Laws:

1. $\dfrac{d}{dx}(c) = 0$

2. $\dfrac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$

3. $\dfrac{d}{dx}\big(cf(x)\big) = cf'(x)$

"Proof"

1. Constant Rule: For $f(x) = c$,

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c - c}{h} = 0.$$

2. Sum rule:

$$\lim_{h\to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h} =$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

3. Constant coefficient rule:

$$\lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h} = c \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$4.\ \frac{d}{dx}(x^n) = nx^{n-1}$$

"Proof"

4. Power Function Rule: For $f(x) = x^n$,

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{x^n + nx^{n-1}h + h^2(\dots) - x^n}{h}$$

$$= \lim_{h \to 0} \frac{nx^{n-1}h + h^2(\dots)}{h}$$

$$= \lim_{h \to 0} nx^{n-1} + h(\dots) = nx^{n-1}$$

Algebra Skills Test

Rewrite each term in the form: $a\,x^b$,

and find the derivative:

1. $y = 2 + 7\sqrt{x^3} + \dfrac{13}{2x^6}$

2. $y = \dfrac{32 \cdot 15\,x^4}{16 \cdot 5\,x^6} + \dfrac{x^7 x^2}{4(x^2)^3} + \dfrac{3}{x}$

3. $y = 17\sqrt[5]{x^3} + 3(2x^5)$

4. $y = x^2(3x^3 - 4x)$

$$5.\ \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

"Proof"

5. Exponential Function Rule:

For $f(x) = a^x$,

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$

$$\text{Note:}\quad \ln(a) = \lim_{h\to 0} \frac{a^h - 1}{h}$$

Find the derivative

1. $y = 5x + 3x^3 + 7e^x$

2. $y = \frac{4(2)^x}{3} - 11 + \frac{8\sqrt[3]{x^2}}{5}$

6. $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$

7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

"Proof"

6. Product Rule:

$$\lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} f(x+h)\frac{g(x+h) - g(x)}{h} + g(x)\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Find the derivatives:

1. $y = x^4 e^x$

2. $y = \dfrac{2x^3}{x^2 + 4}$

3. $y = \left(\sqrt{x} + 4x\right)3^x - \dfrac{14}{x^5}$

4. $y = 6(x + 3)^2 + \dfrac{e^x}{x^3}$